ISSN 1454 - 8003
Proceedings of 2017 International Conference on Hydraulics and Pneumatics - HERVEX
November 8-10, Băile Govora, Romania

LQR CONTROL OF LIQUID LEVEL AND TEMPERATURE CONTROL FOR COUPLED-TANK SYSTEM

Melih AKTAŞ[1], Yusuf ALTUN[2], Oğuz EROL[3]

[1] Duzce University, Engineering Faculty, Electrical and Electronics Engineering, Duzce, Turkey.

[2] Duzce University, Engineering Faculty, Computer Engineering, Duzce, Turkey.

[3] Duzce University, Mechatronics Engineering, Duzce, Turkey.

Abstract: *This paper shows the linear-quadratic regulator (LQR) control of coupled-tank system based on the linearized mathematical model. Although the system is nonlinear, it can be linearized at the operating points. Therefore, the temperature and level controller is designed via the method, which is applicable in practical. Also, the control method is known as an optimal control and it has good performance is obtained by determining some matrices of and designing of control for liquid level and temperature controls.*

Keywords: *Two tanks, level and temperature control, LQR control, optimal control.*

1. Introduction

Liquid tanks which are generally used in industrial facilities have strategic importance because of significance of storage which are highly important for human life. In the process of industrial applications, frequently it is essential to may be store up in tanks and transferred to other tanks as per requirement. It is often necessary to keep the liquid at a certain height or within a certain range[1]. For industrial applications, liquid-level and temperature are important parameter, and widely applied in various field, such as, product tank, water tanks, chemical process systems, liquid storage tanks are important components of lifeline and industrial facilities. The coupled-tank liquid level and temperature controls are typical representative of process control, are hot research topics in control field [2]. The coupled-tank liquid-level control system has nonlinear and complex characteristics, in which the control accuracy is directly affected by system status, system parameters and the control algorithm [3]. Because of that the Linear Quadratic Regulator (LQR) is suitable controller for the coupled tank liquid system. LQR is an optimal controller that requires a state-space linear approximation of the non-linear system but generally has superior performance. LQR measures all states and produces a plant input as a function. LQR stabilizes the system using full state feedback [4]. Because of its characteristics, it is also one of the most important benchmark control problem. The goal of the control is to ensure that the liquid levels in the tanks are maintained at the desired level during the transfer. The coupled tank control systems are a multi-input multi output (MIMO) systems, where input is a control voltage and the output is water level. Tanks have an important place for mixing processes in important industries such as petrochemical industry, paper industry, water treatment industry. Therefore, the liquid level control system is noted in the literature. The control of a nonlinear coupled three tank system is dealt, and the aim is to control the temperature and level of water in tanks by using feedback linearization method [5]. Fractional Order Proportional Integral (FOPI) controllers along with conventional feedforward controllers work better than PI/PID/2DOF-PI/3DOF-PI with feedforward controllers in such situation. FOPI controller is designed using the frequency domain approach. Effectiveness of the controllers is tested to maintain a constant level in the first tank while making the level of the second tank to follow a sinusoidal and square wave reference signals. Experimental results validate the objective of the study [6]. Fuzzy logic control is adopted to liquid tank system which has three coupled tanks together [7]. An adaptive fuzzy control (AFC) system has been proposed to realize level control of two coupled water tanks[8]. Real-time experimental and simulation results with an interval type-II fuzzy logic systems (IT2FLS) are compared with those of a linear quadratic regulator (LQR) for level control of three-tank hybrid system [9]. An observer-based control design has been implemented for a combined four-tank liquid level system [10]. In this paper, LQR control based on optimal control method is designed for the two tanks system.

2. The Coupled-Tank Process Model

The water tank process model [11] is shown in Figure 1. Tank 1 and Tank 2 are coupled as shown. Cold water and warm water can be pumped into the left tank via two control input signals U1 and U2 driving control valves. The flows capacity and the temperatures are Tw, Tc, Qw and Qc, respectively. The flow between the tanks is Qr and the flow out of the outlet valve of tank 2 is Qb. The water levels in the Tank 1 and Tank 2 are H1 and H2, respectively, and the tanks have the same cross-sectional field A. This second valve has the variable opening area Av. Water is rapidly blended in both tanks, and for this reason, it is assumed that the temperature is constant throughout the entire volume of the tank. So, the whole volume is homogeneous in terms of temperature.

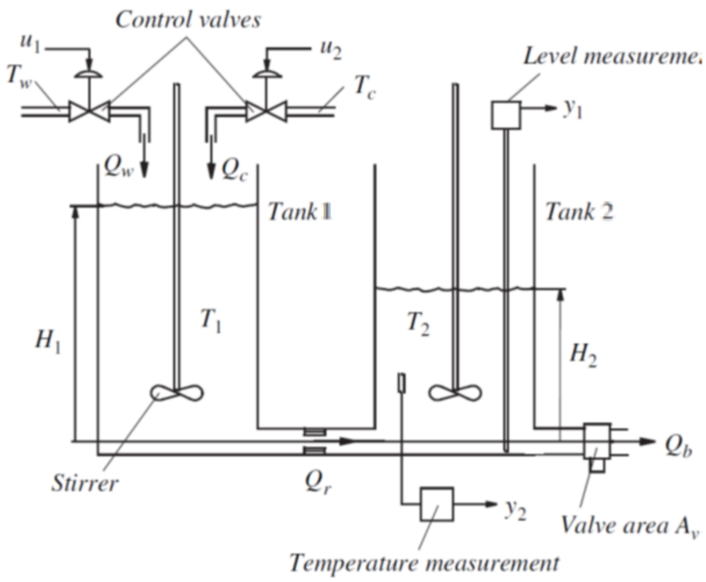

Fig. 1. Water Tank System [11]

Two variables can be measured on the system: the liquid level and the temperature of Tank 2. The measurable variables are as in (1), where k_h and k_t are transducer gains.

$$\begin{aligned} y_1 &= k_h H_2 \\ y_2 &= k_t T_2 \end{aligned} \tag{1}$$

The system equations are as in (2). They are attained from linear equations and therefore they are linearized near a localized working point by the basic linearization technique [11]. Hence, if the left sides of the expressions in (2) are equal to zero, the linear equations are obtained at the stationary operating point. Thus, the linearized state-space equation of the system is as in (3) where Δx (Δx_1, Δx_2, Δx_3, Δx_4) defines ΔH_1, ΔH_2, ΔT_1, ΔT_2, Δu defines Δu_1, Δu_2, and Δv (Δv_1, Δv_2, Δv_3) defines ΔA_v, ΔT_w, ΔT_c. The matrices of state-space are as in (5) and (6). In addition, Δ defines the deviations from the values of stationary. Thus, states are calculated with including deviations in values of stationary as in (4) to the states' working points. As for the system parameters; c is heat capacity of water, k_a is the flow coefficient, ρ is mass density, $D_v = C_d\sqrt{2g}$, A_o is the area of orifice, C_d is a constant loss coefficient and g is gravitational acceleration. In addition, it is assumed that $H_1 \succ H_2$.

$$\begin{aligned}
\frac{dx_1}{dx} &= \frac{1}{A}\left[k_a\left(u_1+u_2\right)-C_0\sqrt{x_1-x_2}\right] \\
\frac{dx_2}{dx} &= \left(\frac{1}{A}C_0\sqrt{x_1-x_2}-D_v\sqrt{x_2}v_1\right) \\
\frac{dx_3}{dx} &= \frac{1}{Ax_1}\left[\left(v_2-x_3\right)k_a u_1-\left(v_3-x_3\right)k_a u_2\right] \\
\frac{dx_4}{dx} &= \frac{1}{Ax_2}\left(x_3-x_4\right)C_0\sqrt{x_1-x_2} \\
y &= \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 & k_h & 0 & 0 \\ 0 & 0 & 0 & k_t \end{pmatrix}
\end{aligned} \tag{2}$$

The added subscript-zero defines the fix operating points.

$$\begin{aligned}
\Delta\dot{x}(t) &= A\Delta x(t)+B_1\Delta w(t)+B_2\Delta u(t) \\
\Delta y(t) &= C\Delta x(t)
\end{aligned} \tag{3}$$

$$H_1 = H_{10}+\Delta H_1(t) \tag{4}$$

$$A=\begin{pmatrix}
\frac{-C_0}{2A\sqrt{x_{10}-x_{20}}} & \frac{C_0}{2A\sqrt{x_{10}-x_{20}}} & 0 & 0 \\
\frac{C_0}{2A\sqrt{x_{10}-x_{20}}} & \frac{-C_0}{2A\sqrt{x_{10}-x_{20}}}-\frac{D_v v_{10}}{2A\sqrt{x_{20}}} & 0 & 0 \\
0 & 0 & -\frac{k_a\left(u_{20}+u_{10}\right)}{Ax_{10}} & 0 \\
0 & 0 & \frac{C_0\sqrt{x_{10}-x_{20}}}{Ax_{20}} & -\frac{C_0\sqrt{x_{10}-x_{20}}}{Ax_{20}}
\end{pmatrix} \tag{5}$$

$$B_1=\begin{pmatrix}
0 & 0 & 0 \\
-\frac{D_v\sqrt{x_{20}}}{A} & 0 & 0 \\
0 & \frac{k_a u_{10}}{Ax_{10}} & \frac{k_a u_{20}}{Ax_{10}} \\
0 & 0 & 0
\end{pmatrix}, B_2=\begin{pmatrix}
\frac{k_a}{A} & \frac{k_a}{A} \\
0 & 0 \\
\frac{k_a\left(v_{20}-x_{30}\right)}{Ax_{10}} & \frac{k_a\left(v_{30}-x_{30}\right)}{Ax_{10}} \\
0 & 0
\end{pmatrix}, C=\begin{bmatrix} 0 & k_h & 0 & 0 \\ 0 & 0 & 0 & k_t \end{bmatrix} \tag{6}$$

3. The Controller Design

LQR is an optimal controller that requires a state-space linear approximation of the non-linear system but generally has superior performance. LQR measures all states and produces a plant input as a function. LQR stabilizes the system using full state feedback. Suppose that state space equations of linear time invariant system is;

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{7}$$

$$y(t) = Cx(t) + Du(t) \tag{8}$$

Performance index of the LQR controller is introduced as follows, where u(t) is input and x(t) is the state of the system.

$$J = \frac{1}{2}\int_0^\infty [x(t)\,\mathrm{Q}\,\mathrm{x}(t) + \mathrm{u}(t)\,\mathrm{R}\,\mathrm{u}(t)]dt \tag{9}$$

J must be minimal in order to achieve an optimal control. Q and R denote the weighting matrix of the state variable and input variable. An optimal control is dependent on Q and R matrix. However, there is no common method in tuning those parameters. Usually simulation trial and error method is used for arranging the correct parameters. However, note that to implement an optimal control an optimal control input must be found u(t). (9) could be written as (10).

$$J = J_0 + \frac{1}{2}\int_0^\infty \left[\left(\mathrm{u(t)} - \mathrm{u_0(t)}\right)' \mathrm{R} \left(\mathrm{u(t)} - \mathrm{u_0(t)}\right) \right] dt \tag{10}$$

Let P be a symmetric matrix, there exists such relation:

$$\int_0^\infty \left[\mathrm{x}' (\mathrm{A}'\mathrm{P} + \mathrm{PA})\,\mathrm{x} + 2x\,\mathrm{PBu} \right] dt = -x(0)' Px(0) \tag{11}$$

By adding and subtracting (11) to (9), (12) is obtained.

$$\mathrm{J} = x(0)' Px(0) + \int_0^\infty \left[\mathrm{x}' (\mathrm{A}'\mathrm{P} + \mathrm{PA} + \mathrm{Q})\,\mathrm{x} + u' Ru + 2x' (\mathrm{PB} + \mathrm{N})\,\mathrm{u} \right] dt \tag{12}$$

Then the optimal control input which minimizes the cost function J is found in (12) by (10) and (11).

$$u_0 = -R^{-1}(B'P + N')x \tag{13}$$

Moreover, Q and R have significant effect on the system performance, if R is large than a smaller input will be applied to stabilize the system. Also, if the error in a certain state needs to be small, the corresponding column of Q needs to be larger. Also keeping Q need and reducing R; results a decrease in transition time and maximum overshoot and an increase in rise time and steady state error.

$$u(t) = -Kx(t) \tag{14}$$

$$K = R^{-1}(B'P + N') \tag{15}$$

The optimal control input is also be found by the help of MATLAB function K =lqr(A,B,Q,R) where K is the LQR gain of the controller. Where the input is being as in (14).

Accordingly, the obtained controller matrix is given by

$$K = \begin{bmatrix} 12.206 & 8.5194 & 17.763 & 12.67 \\ 11.19 & 7.8927 & -9.5695 & -6.7384 \end{bmatrix}$$

4. Simulation Results

For the simulation, Matlab-Simulink is used for the system performance analysis. Table 1 shows that the parameters of the system. According to this, the level and temperature results od second tank are as in Figure 3 and 4 for the initial conditions ΔH_2=0.4m, ΔT_2= 3°C. The references are zero. It means that it is desired that the changes of level and temperature for second tank should be zero. Figure 3 presents the changing of ΔH_2 while Figure 4 presents the changing of output ΔT_2. Regarding the results, the control performance is good response as shown in Figure 2 and Figure 3.

Table 1: The values of parameters for the tank system

The stationary points in the linearization	Value	Parameters	Value
A_{v0}	0.0122m^2	k_h	2 volt/m;
T_{w0}	60°C	k_t	0.1 volt/°C
T_{c0}	30°C		
x_{10}	2.03m	A	0.785 m^2
x_{20}	1.519m	D_v	2.66 m$^{1/2}$/sec
$x_{30}=x_{40}$	45°C	C_0	0.056 m$^{5/2}$/sec
$u_{10}=u_{20}$	5volt	k_a	0.004 m^3/volt.sec;

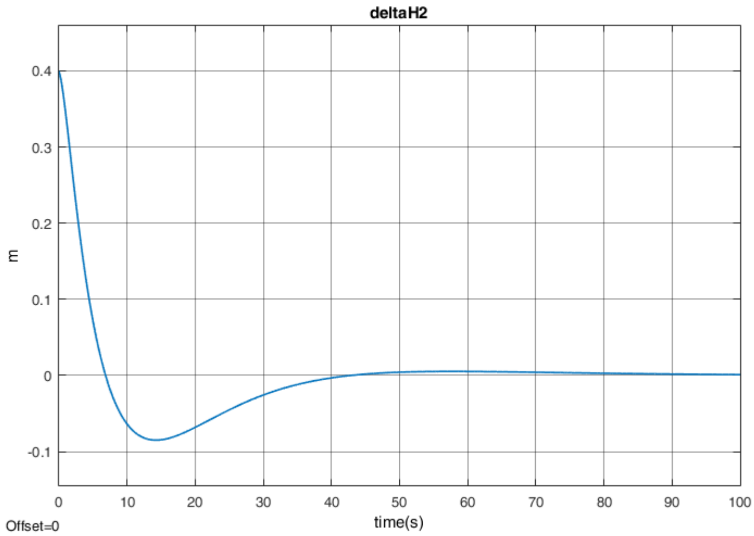

Fig. 2. The level of the tank 2

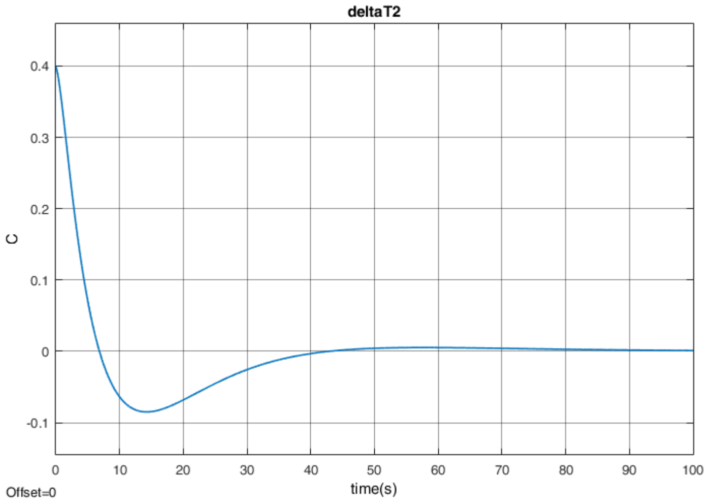

Fig. 3. The temperature of the tank 2

5. Conclusions

This paper shows that LQR control of two water tanks is fulfilled. The controller is designed via optimization software. the controller is an optimal control method. The simulation results show that the controller gives a satisfactory performance according to desired states. The simulation results prove that the controller performance is very good for level and temperature of tank. Finally, there is good settling- time and no overshoot for the two water tank system.

References

[1] H. Abbas, S. Asghar, and S. Qamar, “Sliding Mode Control for Coupled-Tank Liquid Level Control,” *10th Int. Conf. Front. Inf. Technol. (Fit 2012)*, pp. 325–330, 2012.

[2] T. Timur, “Evaluation Of Seismic Behaviour Of Base Isolated Cylindrical Liquid Storage Tanks,” İstanbul Teknik Üniversitesi, 2010.

[3] L. Li, “The application of fuzzy PID controller in coupled-tank liquid-level control system,” *2011 Int. Conf. Electron. Commun. Control. ICECC 2011 - Proc.*, pp. 2894–2897, 2011.

[4] Y. Tekn, “Matlab ve simulink kullanarak lqr ve kutup yerleşimi metotlari ile tepe vinci kontrolü,” 2006.

[5] F. Tahir, N. Iqbal, and G. Mustafa, “Control of a Nonlinear Coupled Three Tank System using Feedback Linearization,” *2009 Third Int. Conf. Electr. Eng.*, pp. 1–6, 2009.

[6] K. Sundaravadivu, V. Jeyakumar, and K. Saravanan, “Design of Fractional Order PI controller for liquid level control of spherical tank modeled as Fractional Order System,” in *Proceedings - 2011 IEEE International Conference on Control System, Computing and Engineering, ICCSCE 2011*, 2011, pp. 522–525.

[7] M. Abid, “Fuzzy logic control of coupled liquid tank system,” in *Proceedings of 1st International Conference on Information and Communication Technology, ICICT 2005*, 2005, vol. 2005, pp. 144–147.

[8] A. Başçi and A. Derdiyok, “Implementation of an adaptive fuzzy compensator for coupled tank liquid level control system,” *Meas. J. Int. Meas. Confed.*, vol. 91, pp. 12–18, 2016.

[9] H. Sahu and R. Ayyagari, “Interval fuzzy type-II Controller for the level control of a three tank system,” in *IFAC-PapersOnLine*, 2016, vol. 49, no. 1, pp. 561–566.

[10] H. Gouta, S. Hadj Saïd, N. Barhoumi, and F. M’Sahli, “Generalized predictive control for a coupled four tank MIMO system using a continuous-discrete time observer,” *ISA Trans.*, vol. 67, pp. 280–292, 2017.

[11] E. Hendricks, O. Jannerup, and P. H. Sørense, “Linear Systems Control,” *Linear Syst. Control*, vol. 0, no. 1, p. 555, 2008.