IMPACT VELOCITY CONTROL OF HOPKINSON BAR MECHATRONICS SYSTEM USING DRIVING ROBUST PRESSURE, NUMERICAL CALIBRATION, STRAIN WAVE MEASUREMENT AND INVERSE ANALYSIS

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Abstract: *This scientific research proposes an improved experimental design together with a hybrid analytical-numerical analysis concerning high-speed pneumatic compression Split Hopkinson Pressure Bars (SHPB) mechatronics system developed at INSA Rennes (France). The main objectives are to describe the entire mechatronic pneumatic propulsion and to analyses the data acquisition. Impact velocity prediction, function of pressure driving and specific set point, elastic deformation shock waves and the proposed new numerical calibration method will be detailed. Experimental measurements and estimation of velocity-pressure curve uses a Hermite cubic splines interpolation method presented together with a numerical analysis of elastic deformation signals from strain gages or Bragg optical fibers. The proposed hybrid techniques use a parametric identification of an analytical model describing the striker propulsion and linear movement taking into account friction effects and a numerical calibration based on a complete finite element (FE) simulation of the entire SHPB device. This takes into account the propagation of elastic shock waves and all dynamic mechanical interactions at the contact interfaces between the launched impact bar at an imposed speed, the incident bar and the sending bar, an elastic-dynamic mechanical equilibrium model is used. Based on previous and current researches at INSA Rennes, as a real application of the proposed full SHPB analytical and FE models, can be shown the material constitutive laws identification using a two-step inverse analysis technique. The identified constitutive equations concern description of thermomechanical materials behaviour subjected to high deformation rates, large plastic deformations and high temperatures, especially when high gradients of all these variables occur during material loadings.*

Keywords: *Pneumatic SHPB Device, Hopkinson Bars (SHPB), Estimated Impact Speed, Hybrid Analytical-Numerical Calibration, Finite Elements Modelling, Numerical Calibration, Inverse Analysis*

1. Introduction

Various manufacturing processes and industrial applications developed in the last decade use metallic or non-metallic structures undergoing high rates of loadings, severe elasto-plastic strains, strain rates and temperatures. Furthermore, localized gradients of strain and stress or complex deformation paths occur during rapid, impact, choc or crash loadings. Modelling the corresponding real material thermos-mechanical behavior becomes a real scientific key. This purpose requires design of specific materials characterizing techniques as dynamic or rapid experimental tests using rigorous experimental calibration of used measurement sensors and reliable data analysis via improved analytical, numerical or hybrid analytical-numerical models. Because very high strain rates occur during these dynamic tests, elastic deformations waves travel along the experimental system bench compounds and specimen material, at very high velocities named celerity, of several kilometers per second. Therefore, it is difficult to have an intuitive understanding of the observed physical phenomena in part caused by strong coupling between different boundaries conditions. To perform the quality measurements and their reliability, during the dynamic deformations obtained from rapid loadings, it is nevertheless necessary to take account the quantitative description of elastic wave propagation. First rapid mechanical tests were conducted around 1870 by John Hopkinson who developed a specific mechanism to impact a cylindrical bar. Bertram Hopkinson [1] introduced in 1914 a pressure bar to can study and reproduce very short dynamic events such as the explosive detonation or the impact of different types of projectiles. The named Hopkinson pressure bar is based on the application of theory concerning elastic strain wave's propagation, to predict strains and stress magnitudes developed in the studied material sample. Hopkinson discovered that the small local displacements in the bar depend directly to the period of the elastic wave obtained during the very short times of the impact caused by the material sound celerity. In the case of materials undergoing an impact through a projectile, Davies [2] shows in 1948 that it is possible to measure the temporal form of the generated elastic wave using strain gauges instrumented bar. The first Split Hopkinson Pressure Bars system (SHPB) was designed in 1949 by Kolsky [3] which has used a gas propulsion device to obtain high speed of a projectile bar. He added two other metallic bars named incident and sending bars to realize a dynamic compression. Generally, the SHPB device can develop impact velocities up to 30-40 m/s and corresponding strain rates in the range of 10^2 to 10^4 s^{-1}. Further, other dynamic experimental set-up has been developed as the Taylor test (gas or explosive propulsion around of 100 m/s), Crossbow system (speed up to 10-30 m/s), traction or torsion Hopkinson Bars, Weight Falling and form last three decades specific hydraulic press using particular actuators and devices to can control the impact velocity (compression or traction speed up to 10 m/s). The main purpose of this scientific work is to describe a mechatronic SHPB compression test designed on GCGM Laboratory of INSA Rennes (France) using a pneumatic propulsion with a robust control of air pressure, a laser optic camera to measure projectile bar speed and automatic electronic data acquisition of bars elastic deformations. Results are presented concerning the use of a specific calibration method developed from a hybrid analytical and numerical finite element system modelling.

2. Pneumatic SHPB Mechatronics System

Kolsky found that the Cauchy stress and the elastic-plastic deformation of an impacted specimen can be directly related to the displacements fields of the incident and sending bars. Contrary of the classical Hopkinson pressure bar, the SHPB device use a projectile does no impact directly the material specimen. It is first an incident bar that receive the impact of the projectile (striker bar), subjected then to a lot of dynamic elastic deformation pulse. The propagation of elastic wave in the incident bars, in the specimen and finally in the sending bar it is more intense as the speed of striker is high and is lasts time longer as the projectile is long. This elastic wave is reflected partially by the material sample on the incident bar, the other part passes through it and is subsequently transmitted to a second bar named transmitter or sending bar [4-7].

2.1 Framework and SHPB design characteristics

The general scheme of mechatronic SHPB system designed and used at INSA Rennes (France) is presented in the Figure 1.

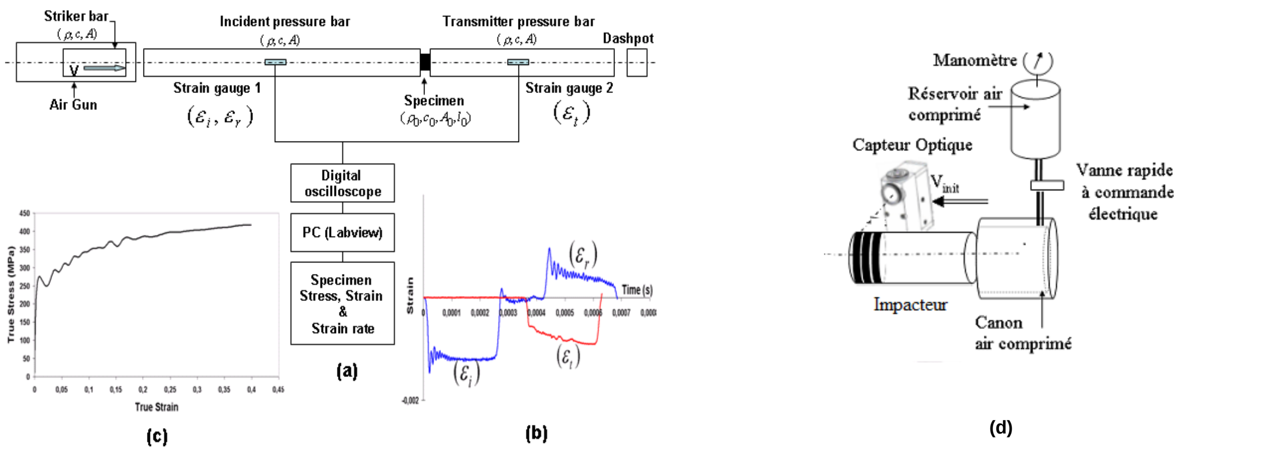


Fig. 1. General schema of the compression Split Hopkinson Pressure Bars (SHPB) bench: pneumatic propulsion bars design with automatic acquisition of laser camera and strain gauges A/B signals (incident $\varepsilon_i(t)$, reflected $\varepsilon_r(t)$ and transmitted $\varepsilon_t(t)$ elastic deformations) performed with a Labwiew program, true stress - true strain curves of material specimen obtained from David code [8].

According to the general theory of elastic wave propagation [4-8], to the type of specimen's materials and to the desired obtained strain and strain rate, the choice of the material and bars geometric size requires to take into account some physical conditions. In a first time it is necessary to have similar elastic impedances of the bars as those of tested material specimens. In particular, to can test steels, aluminium or titan alloys, the hardened high strength steel MARVAL18 is used for bars material (Table 1).

To avoid superposition of measured elastic strain gauges by incident and sending bars [7-8] it is necessary to take into account the minimum measurement time using the following relationships:

$$\Delta t_{measure} = t_i = 2\mathrm{l}_{\mathrm{bimp}} / c_b \le l_{bi} / c_b \text{ and } \mathrm{l}_{bt} / c_b \ge 0.5\mathrm{l}_{bi} / c_b \tag{1}$$

It is then required to have $l_{bi} \ge 2\mathrm{l}_{\mathrm{bimp}}$ and $\mathrm{l}_{bt} \ge 0.5 l_{bi}$. The maximal average plastic strain of material specimen can be estimate from large displacements compression theory by $\bar{\varepsilon}_{\max} \approx \ln\left[1-(2v_{imp}l_{b_{mp}i} / l_0 c_b)\right]^{-1}$. Considering a cylindrical specimen with a length l_0 = 10 *mm* and an initial impact velocity v_{imp} = 10 *m/s* a value of $\bar{\varepsilon}_{\max}$ around of 25%-50% can be obtained for $\mathrm{l}_{\mathrm{bimp}} \in [0.5m, 1m] \Rightarrow l_{bi} \ge 1m \div 2m$ and $l_{bt} \ge 0.5m \div 1m$. More great plastic deformations of specimen, especially for smallest impact velocities, can be obtain for shorter specimens or specimens with special or unconventional local shape as dumbbell sample or hat one. To minimize effect of the radial dispersion elastic waves and to have bars elastic deformations conditions close to the infinite wave propagation theory [7], the bars diameters d_b must to be very small as compared to the twist of striker bar lengths i.e. $d_b \ / \ c_b t_i \ = \ d_b \ / \ 2l_{bimp} \ll 1$. The Table 1 and Table 2 synthetize the chosen material and geometric characteristics of INSA Rennes Hopkinson bars in order to perform compression impact tests with large plastic deformations of material specimen at high strain rates (more than 500s^{-1} – 1000s^{-1} corresponding to $v_{\mathrm{imp}} \in [10m/s, 30m/s]$.

Table 1: Material and geometric characteristics of SHPB bars (total length of bench support ≈ 5.5 m)

Bars	Air Gun	Striker Bar	Receiving Bar	Sending Bar
Material	MARVAL 18	MARVAL 18	MARVAL 18	MARVAL 18
Diameter [mm]	ϕ_{in} 30.0 and ϕ_{out} 40.0	ϕ 16.0	ϕ 16.0	ϕ 16.0
Length [m]	2.0	0.602 (0.5÷1)	2.0	1.3

Table 2: Elastic mechanical properties of MARVAL 18 steel bars

Temperature 20°C	E_b [GPa]	υ	$R_{0,2}$ [MPa]	ρ_b [Kg /m^3]	$c_b = \sqrt{E_b / \rho_b}$ [m/s]	$Z_b = \rho_b c_b = \sqrt{\rho_b E_b}$ [Kg/m^{2}s]
MARVAL 18	186	0.33	1840	8000	4821.82	38.57·10^6

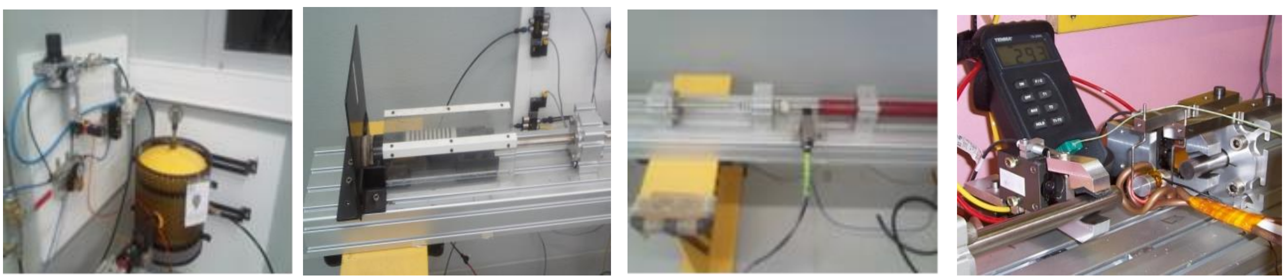


Fig. 2. View of experimental pneumatic SHPB bench with presentation of principal pneumatic propulsion system, control, measurement devices and inductive heating

As one can see in the Figure 2, the striker bar is driven by a pneumatic device using a tank with a volume V_0 of *20* l coupled on a pressurized air with maximum 10 bars via a circuit of rapid control, regulate and secure valves. Using a lot of plastic cylindrical PFTE collars mounted on the striker bar, this one is moved in axial translation by the air pressure propulsion on a distance of *1 m* inside the gun with 2 m length undergoing a small friction effect. The axial impact velocity is measured by a laser optic camera reading a barcode of *50 mm* large with a length of $\lambda_v = 5mm$ for each uniform distributed white/black slots via a paper barcode glued on the end of striker bar surface. The initial impact speed value is obtained by division of the slot length λ_v with the corresponding measured period times T_v recorded from a specific Labview program coupled on a high speed NI PCI 6110 acquisition card of 5 *MHz*. The Labview interface coupled to a digital VISHAY conditioner is also performed to record in Volts the experimental signals corresponding to two full strain bridges mounted on the half part of the incident bar and half part of the sending bar using a digital conditioner VISHAY (Figure 1). So the incident $\varepsilon_i(t)$, reflected $\varepsilon_r(t)$ and transmitted $\varepsilon_t(t)$ elastic strain waves of the bars can be estimated either through from a direct conversion using strain bridge formula or directly from a numerical calibration techniques. Computation of bars/specimen interfaces velocities and loads can be performed from analytical relationships based on the general dynamic elastic deformations propagation inside the bars [7] using David software [8]. A conventional thermocouple or an infrared camera coupled to an inductive heating coil together is used together with a thermal control device. The inductive heat system is then able to keep the initial metallurgical structure of metal specimen and to can estimate experimentally the material self-heating. It is then possible to perform impact compression tests for different initial temperature of material specimen. The identification of the thermo-mechanical material specimen behaviour in terms of true stress - true elasto-plastic strain for different strain rates and initial temperatures can be make using plasticity theory, hybrid numerical-analytical methods and inverse analysis strategy [9-14].

2.2 Pneumatic control circuit and experimental data acquisition

The propulsion control system of designed SHPB bench is fully pneumatic and in open loop as shown in the Figure 3. When the set pressure of tank air is reach, the operator triggers manually the air gun to trip the axial propulsion of the striker bar.

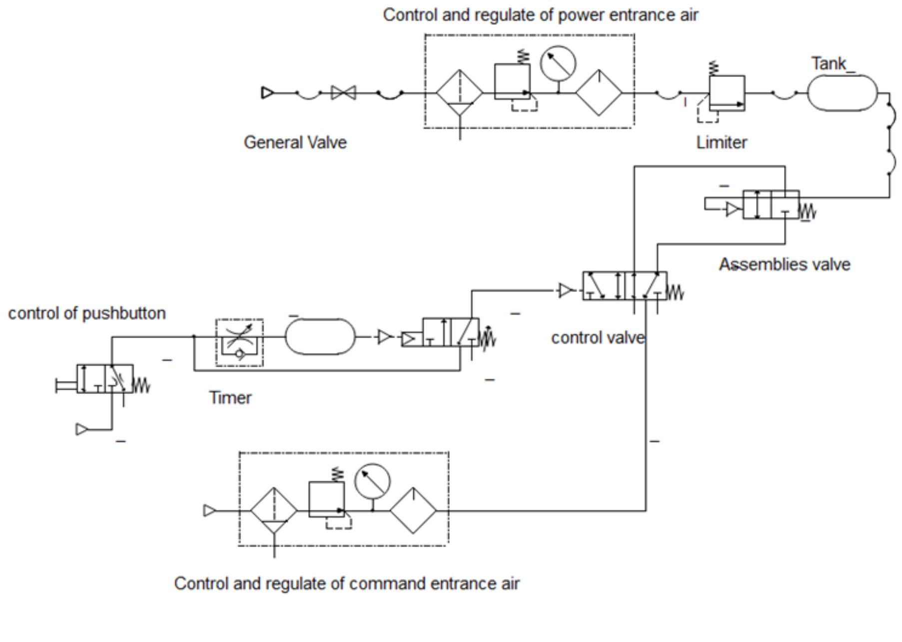
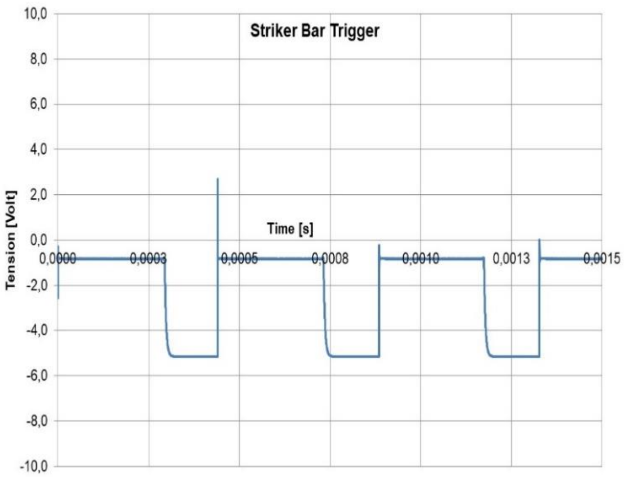
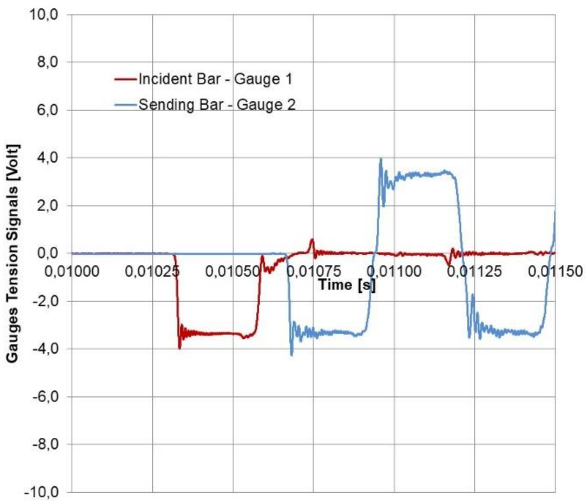


Fig. 3. Pneumatic control circuit used for propulsion of experimental mechatronic SHPB bench [16]

However, the implementation of a digital control system with feedback of different states of the system will improve its control allow precise and constant pressures when start the move of the striker bar inside the air gun. To resume, with an automatic control system, the pressure keeps constant value on the time gap between value pressure regulation and the launch of the test, the repeatability and robustness of the experiments guaranteed and more rigorous studies will be possible. After starting and triggering of the rapid control valve, the air gun projects the striker against the receiving bar. Before the shock, the laser sensor captures on the front of the incident bar the variation of a signal from an uniform succession of black and white slots with a length of λ_v. Thus is generate a square-shaped signal of period T_v, where the first rising edge acts as a trigger for optical sensor acquisition. The impact speed is deduced there from the ratio λ_v / T_v. The Labview language program developed for a real time acquisition of the optic sensor and gauge full bridges raw signals allows adjustment of the acquisition frequency f_a and of the scanning number b_s. The product $f_a \cdot b_s$ determines the total acquisition duration. In a first time to can calibrate the experimental bench, an experimental impact test without any specimen between the bars it realized. The Figure 4 plot the recorded experimental tensions obtained from the used laser camera reading the striker barcode with $\lambda_v = 5\,mm$ and from two full gauge bridges glued on incident and sender bars in order to measure the corresponding elastic wave strains for an air tank pressure around of 1.9-2 bars and an acquisition frequency f_a = 1 *MHz*.



a)



b)

Fig. 4. a) Square-Shaped Dirac tension signal of laser optical camera recorded by triggering option using Labview program, b) Gauges tension signal variation corresponding to the elastic wave deformation of the incident bar ($\varepsilon_i(t)$, $\varepsilon_r(t)$ for red curve) and sending bar ($\varepsilon_t(t)$ for blue curve) corresponding to a set tank air pressure of 1.9 bars - 2 bars [16]

As can be shown in Figure 4 a) the initial impact velocity can be estimate from the square-shape signal corresponding to the succession of black and white slots with a constant length of λ_v = 5 mm in the time period T_v estimate to be equal to 0.5138 *ms* by $v = \lambda_v / T_v \approx 10\,m/s$. Concerning the Figure 4b), in a classical way the full bridge gauge tension *U* (μV) is expressed in function of the corresponding elastic deformation value ε (μdef) starting from the formula:

$$U = U_0 F_g \varepsilon (1+\upsilon) / \left[2 + f_g \varepsilon (1-\upsilon) 10^{-6} \right] \qquad (2)$$

where U_0 is the bridge supply voltage (expressed in Volt) and f_g the gauge factor given by the strain gauge. Starting from an acquisition frequency of 1 *MHz* and from the very short time of the impact period around of 1 *ms* it is required to set the VISHAY conditioner to record 1μ*V*/μ*def*. To obtain a reliable sensitive recorded tension, the dynamic conditioner amplifier uses a gain factor *G* i.e. $\hat{U} = GU$ and the equation (6) gives:

$$\varepsilon = 2\hat{U}\cdot 10^{6} / \left[U_0 G F_g \left(1+\upsilon\right) - \hat{U} F_g \left(1-\upsilon\right)\right] \approx 2\hat{U}\cdot 10^{6} / U_0 G F_g \left(1+\upsilon\right) \qquad (3)$$

assuming $U_0 G \gg \hat{U}$ i.e. $U_0 \gg U$ where $\hat{U}$ (expressed in Volt) represents the tension values recorded by the output of Labview program.
According to the elastic strain gauge principle the variation of gauge deformation with the recorded tension is quasi-linear, consequently a calibration factor K_{exp} can be defined starting from the ratio between the corresponding gauge deformation value and the recorded tension i.e.:

$$K_{exp} = \varepsilon / \hat{U} \quad (\mu def / Volt) \qquad (4)$$

For the used strain full bridge, the gauge factor f_g is equal to 2.09, the gain *G* is set around of 250 and the supply voltage of the bridge is chosen to be 7.5 *V*. Consequently, it is obtained a calibration factor K_{exp} = 1000/2.6 = 384.62 μ*def /Volt*.

2.3 Experimental estimation of initial impact velocity using Hermite interpolation method

The cubic Hermite spline method is an interpolation method by parts based on cubic Hermite polynomials. This method derives a third order polynomial of Hermitian form for each defined interval and only guarantees continuity for the first derivatives of polynomials interpolation. The cubic Hermite method has more local property than the classical cubic spline method. That is, if you change a data point xj, the effect on the interpolation result it is in the range defined by [x_{j-1}, x_j] and [x_j, x_{j+1}].

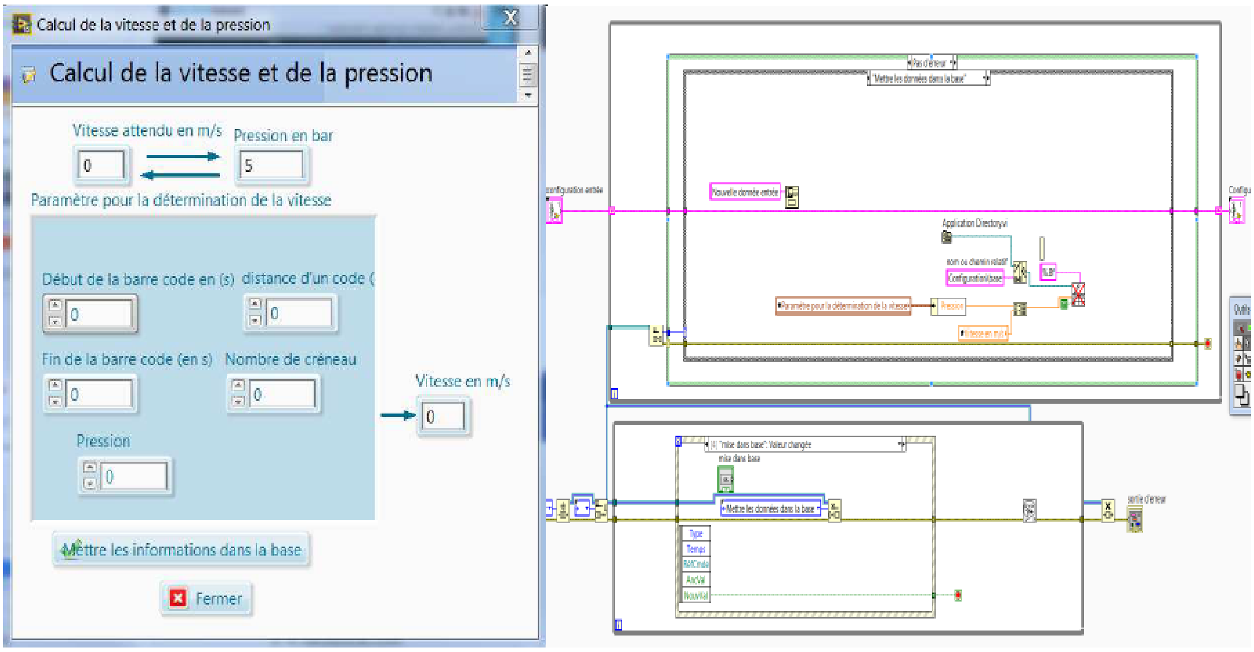


Fig. 5. Labview User Interface to set air pressure function of a predefined initial impact velocity

If known n experimental points, the Hermite polynomials P(x) of degree 2n+1 defining the function f variation is defined by

$$P_i(x) = f(x_i) + (x - x_i)\left[f'(x_i) - q'_i(x_i) f(x_i)\right],\ P(x) = \sum_{i=0}^{n} q_i(x) P_i(x) \text{ and } q_i(x) = \prod_{j=0,\, j\neq i}^{j=n} \left(\frac{x - x_j}{x_i - x_j}\right)^2 \qquad (5)$$

Here *f* represents here the velocity variable and *x* the pressure value. *f'* is computed by a finite difference method starting from previous measured values. If coupling between Matlab and Labview it is more reliable to use the predefined function of cubic Hermite interpolation „*Pchip*" of Matlab.
Concerning the use of SHPB system, the main purpose is to determine the tank air pressure according to the desired striker speed starting form a lot of previous experimental values and using interpolation based on the above Hermite functions *P(x).* On the basis of SHPB tests which have already been carried out, the user can choose to add each new measured velocity to the built base in order to improve interpolation quality and to obtain an auto-learning strategy. Using Labview program the Figure 5 show the obtained auto-learning user interface. Based on a lot of 15 experimental measurements of striker impact speed using the laser optic camera for a set of air pressure in the range of 1.6 bars – 2.9 bars, the Hermite interpolation and Labview interface find the velocity-pressure curve diagram pictured in Figure 6.

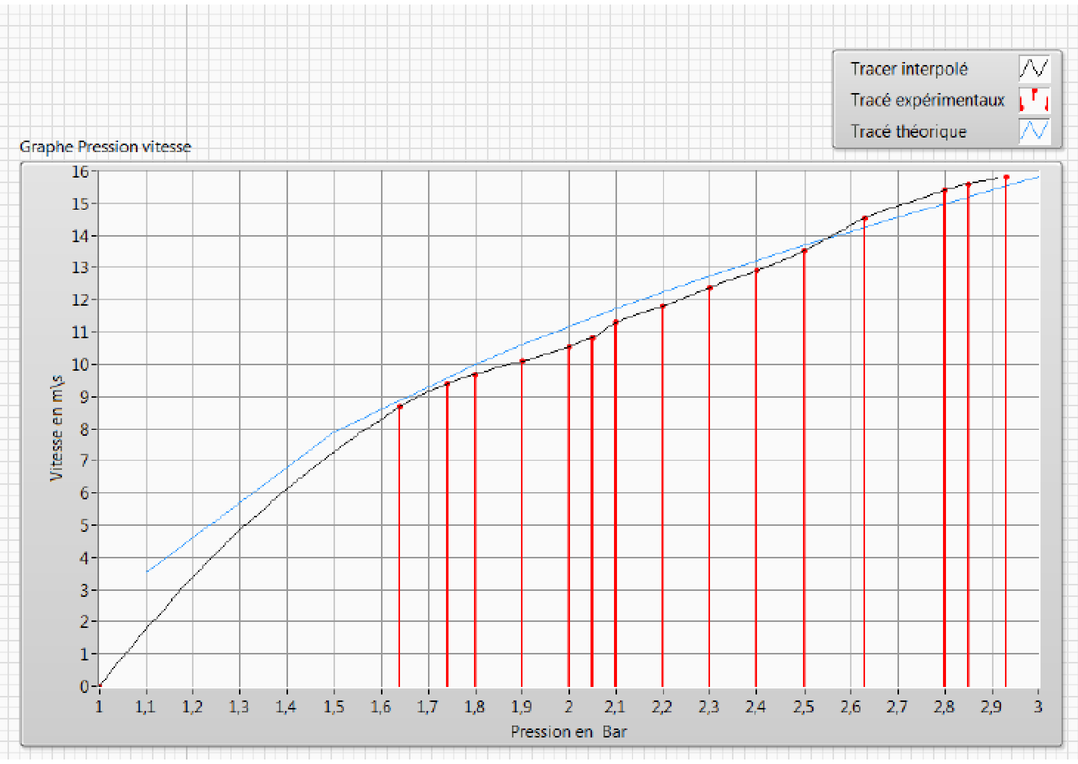


Fig. 6. Hermite interpolation of striker impact velocity – air pressure curve

Using this method, the user can estimate the air pressure choice corresponding to a specific initial impact velocity chosen in the range of 9 m/s – 15 m/s. This method can be use if experimental SHPB test not need precise initial impact speed. Therefore, regarding that the extrapolation outside the diagram range is too approximate and very imprecise a more robust control of striker impact velocity is need based on validation of a proposed reliable numerical calibration method.

2.4 Theoretical estimation of initial impact velocity

To be able to obtain a purely axial translation of the striker this one have a number of 5 glued cylindrical PTFE collars with a total length of $L = 80 \times 10^{-6}$ *m*. Using theorem of mechanical energy balance the sum of kinetic energy variation of the striker ΔE_c and friction energy $E_f = \int_0^l \Delta E_f dx$ is equal to the work *W* of the bar surface force generated by air pressure i.e.:

$$\Delta E_c + E_f = \frac{1}{2}mv^2 + E_f = W = \int_0^l \left(\iint p(x) dS \right) dx \text{ with } m = \rho g l_{imp} \pi d_b^2 / 4 \tag{6}$$

where m is the mass of the striker bar, v the impact velocity obtained after the move on a distance equal to l, $p(x)$ represents the air pressure value of each axial bar position x *varying between 0 and* l and ΔE_f is the specific friction energy computed from an infinitesimal cylindrical slice with diameter D and length dx' corresponding to the collars contact.

Or if p_c is the initial absolute pressure inside the tank of volume V_0, taking into account the atmospheric pressure p_0 and the isothermal perfect gas law, the axial pressure variation is obtained from:

$$p(x)\left(V_0+Sx\right)=p_rV_0 \text{ with } p_r = p_c - p_0 \tag{7}$$

Assuming a uniform pressure distribution on the plastic collar glued on the striker bar along its section S, the corresponding mechanical work of pressure can be computed by:

$$W=\int_0^l p(x)Sdx=\int_0^l\left(\frac{p_rV_0S}{V_0+Sx}\right)dx=p_rV_0\ln\left(1+\frac{lS}{V_0}\right) \tag{8}$$

Concerning the contact between the collars of striker bar and inner surface gun, if supposed to have only smallest elastic deformation, a Coulomb friction law can be used and the corresponding friction energy can be computed by:

$$\Delta E_f=\int_0^L F_f dx' \text{ with } F_f=\mu\pi Dn_f \tag{9}$$

Here D is the collars diameter equal to the inner gun circular section diameter, n_f represents the specific normal force applied to the slice collars with dx' length and μ represent the Coulomb friction coefficient. Using axial mechanical equilibrium of collar infinitesimal slice with a length dx', supposing elastic incompressible cylindrical compression of collars and striker under the action of the axial pressure p(x), it can be written:

$$\sigma_{rr}(x')=n_f(x'),\sigma_{zz}(x') \text{ with } x'\in\left[x,x+dx'\right] \text{and} \, \sigma_{zz}(x)=p(x) \tag{10}$$

$$\left[\sigma_{zz}(x+dx')-\sigma_{zz}(x)\right]\pi D^2/4=\left[dn_f/dx'\right]dx'\pi D^2/4=\mu n_f dx'\pi D \tag{11}$$

where σ_{rr},σ_{zz} are respectively the radial and axial stresses.

Consequently, it can be expressed the specific normal force by:

$$n_f=p(x)\exp\left(-4\mu x'/D\right) \tag{12}$$

Finally using equation (9) the friction energy is given using following expression:

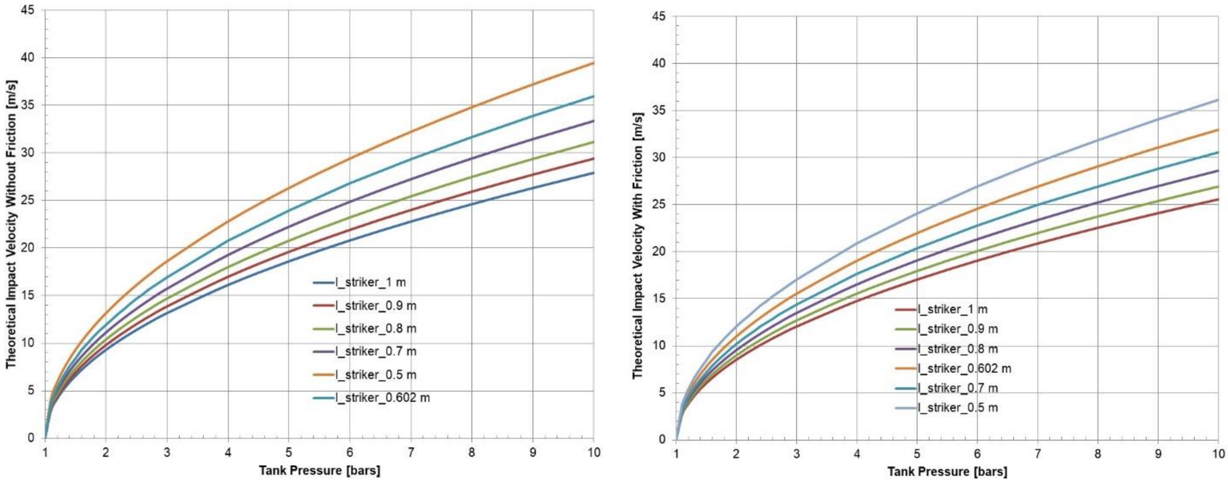
$$\Delta E_f=\left(\pi D^2/4\right)p(x)\left[1-\exp\left(-4\mu L/D\right)\right] \text{ and } E_f=W\left[1-\exp\left(-4\mu L/D\right)\right] \tag{13}$$

Starting from Eq. 6 the obtained impact velocity can be obtain from the following relationship:

$$v=\sqrt{\frac{2V_0}{m}\ln\left(1+\frac{lS}{V_0}\right)\exp\left(-4\mu L/D\right)}\cdot\sqrt{(p_c-p_0)} \text{ or } v=\left(\alpha'/\sqrt{l_{imp}}\right)\sqrt{(p_c-p_0)} \tag{14}$$

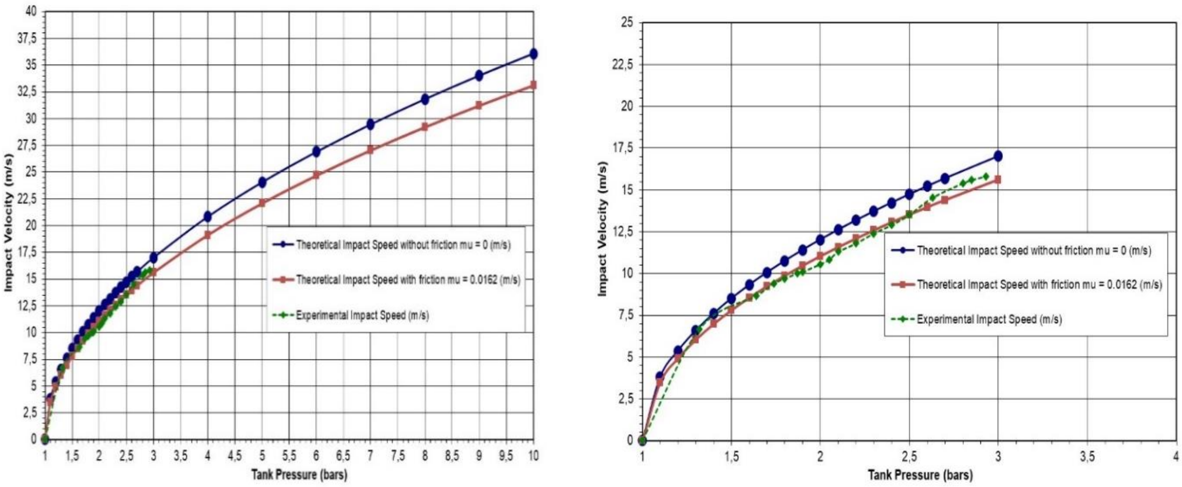
Here α' is a variable depending of tank capacity volume V_0, gun inner diameter D and section $S=\pi D^2/4$, friction coefficient μ, total collars length L and sticker material density ρ. Starting from the SHPB material properties and geometric characteristics given on Table 1 and 2 the tank volume is $V_0=20\,l=2\cdot10^{-2}m^3$, the displacement is l=1m and the collar surface glued on the front of the striker bar has a surface $S=\pi D^2/4=\pi\cdot30^2\times10^{-6}/4\,m^2=706{,}86\times10^{-6}m^2$ which approximate $\ln\left[1+(lS/V_0)\right]=0{,}035$ and $\alpha'=\alpha\approx0.029$ if neglecting friction phenomena i.e. $\mu=0$ as can be considered in a previous research work [16,17].

Using a numerical calibration strategy starting from experimental striker velocities measured by optic camera, a value of μ = 0.0162 is obtained solving a parameter identification problem based on $\underset{\mu}{Min}\left[\left(\sum_{i=1}^{N\exp}\left(v-v^{\exp}\right)^{2}\right)/\sum_{i=1}^{N\exp}v^{\exp^{2}}\right]$ and using a non-linear least squares Levenberg-Marquard algorithm of Matlab software. The identification error of friction coefficient is around of 3%. Regarding the experimental values of a Coulomb PTFE/Steel friction contact, the scientific literature shows a friction coefficient value around 0.02 close to the obtained identified value. Regarding the relationship (14), if friction term is taking into account it is obtain a factor $\alpha' \approx 0.027$. Then, it can be concluded that the estimation error of initial impact velocity is around of 7%- 8% between the case neglecting friction phenomenon and the case taking into account a Coulomb contact friction. Impact velocities estimations for different striker bar lengths l_{imp} from 0.5 *m* to 1 *m* are plot in Figure 7.



a) b)

Fig. 7. a) Curves variation of Theoretical Impact Velocities – Tank Pressure for different striker bar length without friction phenomena (0.5 m to 1 m), b) Curves variation of Theoretical Impact Velocities – Tank Pressure for different striker bar length using friction phenomena with μ = *0.0162* (0.5 m to 1 m)



a) b)

Fig. 8. Comparison between experimental and theoretical impact speed values obtained for the striker with a length of 0.602 m a) without friction (μ = *0*) and b) taking into account the friction (μ = *0.0162)*

One can observe that using a maximal tank pressure of 8 bars for striker bars with short length (0.5 m to 0.6 m) the impact velocity varied from 5 *m/s* to 30-35 *m/s* as compared to an impactor with a length of 1 *m* where the maximum impact speed is limited to 25 *m/s*. The use of a striker with the length of 0.602 *m* (mass $m \approx 0,96\,Kg$), if the tank pressure is expressed in bars an estimation of impact velocity can be obtained by $v \approx 11,82\sqrt{(p_c - p_0)}$ if friction is neglected and by $v \approx 10,84\sqrt{(p_c - p_0)}$ if a friction coefficient μ = *0.0162* is used. Figure 8 shows the comparison between the experimental and theoretical impact velocities.

Table 3: Theoretical and experimental impact speed obtained from different values of tank pressure for the striker with a length of 0.602 m

Tank Pressure (bars)	Theoretical Impact Speed Estimation Without Friction (m/s)	Theoretical Impact Speed Estimation Using Friction (m/s)	Tank Pressure (bars)	Exp. Impact Speed (m/s)
1	0	0	1	0
1.1	3.80	3.49	-	-
1.2	5.38	4.93	-	-
1.3	6.59	6.04	1.32	6.68
1.4	7.61	6.98	1.55	6.86
1.5	8.51	7.80	1.64	8.67
1.6	9.32	8.55	1.74	9.41
1.7	10.06	9.23	1.8	9.7
1.8	10.76	9.87	1.87	10
1.9	11.41	10.47	1.9	10.1
2	12.03	11.03	2	10.56
2.1	12.62	11.57	2.05	10.84
2.2	13.18	12.09	2.1	11.3
2.3	13.72	12.58	2.2	11.8
2.4	14.23	13.06	2.3	12.4
2.5	14.73	13.51	2.4	12.9
2.6	15.22	13.96	2.5	13.52
2.7	15.68	14.39	2.63	14.54
3	17.01	15.60	2.8	15.4
4	20.84	19.11	2.85	15.6
5	24.06	22.07	2.93	15.82
6	26.90	24.67	-	-
7	29.47	27.03	-	-
8	31.83	29.19	-	-
9	34.03	31.21	-	-
10	36.09	33.10	-	-

The experimental values of initial impact velocities measured by laser camera for a lot of tank pressures show very good correlation with the curve of theoretical variation plotted in Figure 8b and values detailed in Table 3. An average estimation error of 8 % is obtained due essentially to the friction phenomena caused by the local contact of the guidance glued collars of striker bar with the inside surface of the gun.

3. Numerical Analysis

To make the SHPB mechatronic system analysis in both experimental and numerical point of view the Figure 9 pictures the flowchart of proposed integrated design strategy, robust control and identification of tested material specimen undergoing a thermo-mechanical compression impact.

Fig. 9. Flowchart of proposed hybrid analytical-numerical analysis of SHPB system using numerical calibration, entire SHPB Finite Element Modeling and Inverse Analysis

3.1 Numerical Calibration and Finite Element Modelling of SHPB System

To improve the experimental calibration procedure in order to avoid approximations of given gauge factor, gain choice and measurements errors, a robust and more general numerical method it is proposed. This numerical calibration method is based on a lot of empty SHPB experimental tests without specimen performed for different air tank pressure and consequently different initial impact velocities, using a Finite Element Modelling of the entire bar's system following the below steps:

I. Empty experimental compression SHPB test (without any specimen) with direct contact of incident and sending bar and at a set of initial impact velocity *v* chosen with respect to the above impact velocity-pressure diagrams. It can be proven theoretically that $\varepsilon_i < 0, \varepsilon_r = 0, \varepsilon_t = \varepsilon_i < 0$ [14];

II. Recorded of tensions time variation $\widehat{U}_i(t)$, $\widehat{U}_r(t)$ and $\widehat{U}_t(t)$ (corresponding to the experimental gauge elastic deformations $\varepsilon_i(t)$, $\varepsilon_r(t)$ and respectively $\varepsilon_t(t)$ found using Labview program);

III. Estimation of the real initial impact speed $\hat{v}$ from the time period T_v of the first square-shaped time Dirac signal of the recorded laser optic camera measurements using the formulas $\hat{v} = \lambda_v / T_v$;

IV. Estimation of the experimental celerity c_{exp} from the time period $t_{i_{\exp}}$ corresponding to the first slot of recorded incident signal $\widehat{U}_i(t)$ using the formulas $c_{\exp} = 2l_{imp} / t_{i_{\exp}}$;

V. Finite Element Simulation of the entire SHPB system using same initial and boundaries conditions as the experimental one (same celerity speed value and same initial impact velocities) with extraction of elastic strains time variation $\varepsilon_{i_{num}}(t)$, $\varepsilon_{r_{num}}(t)$ and $\varepsilon_{t_{num}}(t)$ corresponding to the geometric points positions of the two gauge bridges (one on the half part of the incident bar and other on the half part of the sending bar).

VI. Computation of the numerical calibration factor $K_{num} = \mathrm{Max}\left|\varepsilon_{i_{num}}(t)\right| / \mathrm{Max}\left|\widehat{U}_i(t)\right| = \hat{\varepsilon}_{i_{num}} / \widehat{U}_i$

VII. Experimental – Numerical Comparisons of the time variation concerning incident, reflected and transmitted elastic strains.

An axisymmetric dynamic Finite Element Modelling of SHPB test choosing an initial impact velocity of 10 *m/s* and an incremental time of 10^{-6}s is performed using Cast3M [18], ABAQUS and LS-Dyna code [9-13, 14-15] based on tridimensional elastic properties of the bars, inertial effect and QUAD4 mesh (Figure 10).

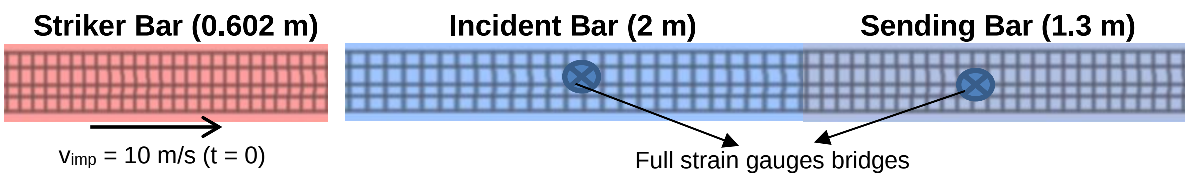
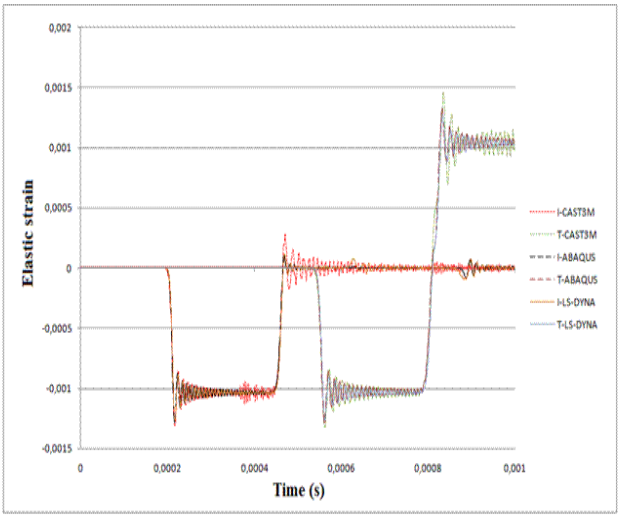
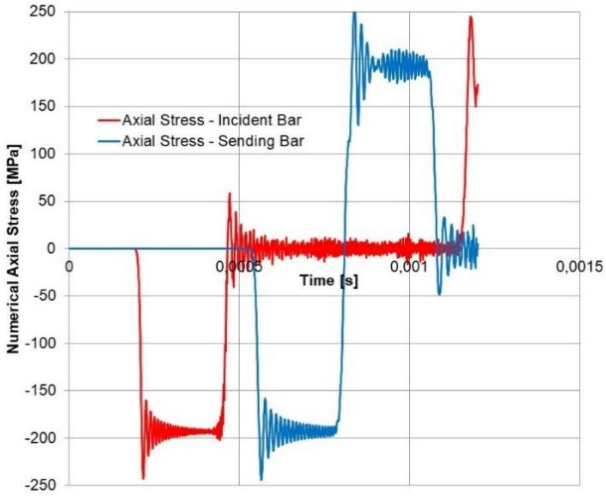


Fig. 10. Mesh of the striker, incident and sending bars used for a Dynamic Finite Element Simulation of the SHPB device using Cast3M, Abaqus and LS-Dyna code

Results concerning the elastic deformations and axial stress obtained from gauge bridges positions are illustrate in Figure 11.



a)



b)

Fig. 11. Numerical results corresponding to the strain gauge bridges position obtained from the Dynamic Finite Element Simulation using Cast3M, ABAQUS and Ls-DYNA code a) Elastic Strains, b) Axial Stress

An analytical estimation of calibration factor K_{an} can be obtained from the ratio of values corresponding to numerical incident elastic deformation $\hat{\varepsilon}_i$ and measured tension $\widehat{U}_i$:

$$K_{an} = \hat{\varepsilon}_i / \widehat{U}_i \ (\mu def / Volt) \tag{15}$$

Or the general theory of bar's elastic wave propagation shown that for an impact velocity v $\hat{\sigma}_i = 0.5\rho c_b v \cong E_b \hat{\varepsilon}_i$ and $\hat{\varepsilon}_i = \rho c_b v / 2E_b = v / 2c_b$. So:

$$K_{an} = v \cdot 10^6 / 2c_{\exp} \hat{U}_i \ (\mu def / Volt) \tag{16}$$

Here the bar's celerity c_{exp} can be estimated using the expression (1) from the time period $t_{i_{\exp}}$ of the first slot of the recorded incident signal $\widehat{U}_i(t)$ by $c_{\exp} = 2l_{imp} / t_{i_{\exp}}$. The signal of incident deformation measured in Volts has a value close to $\hat{U}_i = 3{,}33V$ with a time broadness $t_i = 0{,}254\,ms$ and the experimental celerity can be evaluated as $c_{\exp} = 2l_{imp} / t_{i_{\exp}} = 2 \cdot 10^3 \cdot 0{,}602/0{,}254 \approx 4740\ m/s$. As compared to a previous estimation about of 4821 *m/s* [11] the error of the sound celerity is smallest that 2.5%. Using the equation (10), the analytical calibration factor becomes K_{an} = 317 *μdef /Volt* i.e. 17% differences as compared to the previous experimental or classical strain gauge's calibration factor i.e. K_{exp} = 384.62 *μdef /Volt*. From Fig. 8 it can be observed that $\mathrm{Max}\left|\varepsilon_{i_{num}}(t)\right|$ = 1034.7 *μdef* and $\mathrm{Max}\left|\sigma_{i_{num}}(t)\right|$ = 191 *MPa* values close to the analytical estimations given by $\hat{\varepsilon}_{i_{an}} = v / 2c_b \cong$ 1054.9 *μdef* and $\hat{\sigma}_{i_{an}} = 0{,}5\rho c_b v \cong$ 189.6 *MPa*. Taking into account the experimental tension value obtained in Volts from the incident deformation signal, the numerical calibration factor can be estimated by $K_{num} = \widehat{\varepsilon}_{i\_num} / \widehat{U}_i = 310{,}7\mu def\ / Volt$. As compared to the analytical value, the error is around of 2%. Despite the validation of the proposed calibration method it is possible to conclude that the numerical calibration strategy is more robust and can be performed for more complex conditions as for example in the case of viscoelastic or non-metallic materials bars, or for optic fibbers measurements of elastic deformations where analytical computation models are too approximate or no more valid.

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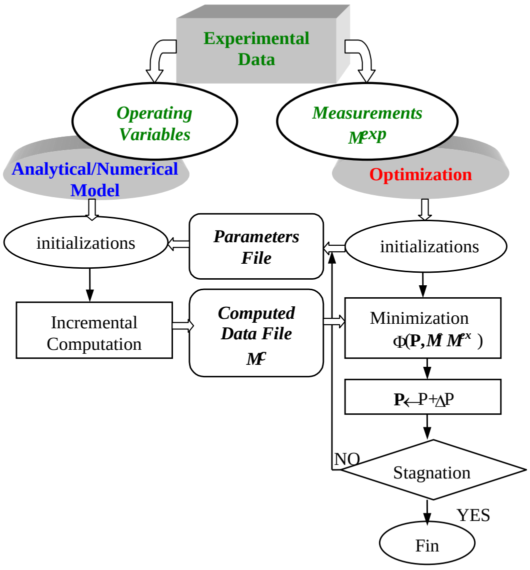
3.2 Numerical Inverse Analysis of SHPB System

The entire Finite Element Model has been performed to simulate SHPB compression tests using different shape of specimens (dumbbell or hat cylindrical samples) undergoing complex strain path especially used to identify by a Two-Step Inverse Analysis strategy the non-linear thermo-mechanical material constitutive equations and the corresponding material coefficients. Details can be show in previous works of Gavrus et al. [9-17]. The proposed Two-Step Inverse Analysis consists in a first step to compute the specimen interfaces velocities and loads starting from the solution of an Analytical Inverse Problems based on equations of elastic wave propagation and on the measured strain gauges measurements followed by a second Inverse Analysis at the specimen scale using Finite Element Modelling of sample elasto-plastic deformations. Finally, the experimental strain gauges signals are compared to the computed ones using Finite Element Simulations of entire SHPB System to valid the used material constitutive law identification method pictured in Figure 12.



$$\Phi(P)=\frac{\sum_{k=1}^{N\exp}\left[M^{c}(P)-M^{ex}\right]^{2}}{\sum_{k=1}^{N\exp}\left[M^{\exp}\right]^{2}}$$

or

$$\Phi(P)=\sum_{k=1}^{N\exp}\left[\frac{\left(M^{c}(P)-M^{ex}\right)}{M^{ex}}\right]^{2}$$

GAUSS-NEWTON OR LEVENBERG-MARQUARD MINIMISATION ALGORITHM

$$[\Delta P]=-\left[\frac{d^{2}\Phi}{dP^{2}}\right]^{-1}\left[\frac{d\Phi}{dP}\right]$$

Identified Parameters P

Fig. 12. Inverse Analysis Principle applied to solve Non-Linear Identification or Inverse Problems

4. Conclusions

The above experimental design and quantitative description of the pneumatic compression mechatronic SHPB bench confirms the robustness of impact speeds control together with strong validations of their theoretical dependency on tank air pressure set. A new hybrid analytical-numerical calibration method was detailed in order to can estimate the elastic wave strains which travels with a specific celerity the incident and sending bars. A complete dynamic Finite Element Modelling and Simulation of the SHPB system without specimen has been performed to establish a more rigorous estimation of the conversion factor between the measured gauge full bridge tension and real elastic strain value. Comparisons with analytical formula based on elastic wave propagation theory of infinite bars have shown the high precision of the Finite Element Modelling results, which permit to valid the entire calibration strategy. It is also possible to confirm again the rightness of the proposed two-step Inverse Analysis technique developed from 1998 during previous research works at INSA Rennes to identify specific constitutive equations of metallic materials under severe loadings and complex deformation paths: large plastic strains, high strain rates, temperature influence and important local gradients of thermo-mechanical variables. Regarding the generality of the proposed numerical calibration method, this one will be apply in a future research work to improve the SHPB acquisition system by use of local Bragg optic fibers sensors and specific optical interrogators developed by Dimione Systems of France and Redondo Optics Company of USA to measure in a more accuracy manner the elastic deformations of the bars.

References

[1] Hopkinson, B. “A method of measuring the pressure produced in the detonation of high explosives or by the impact of bullets.” *Philosophical Trans. of the Royal Society of London* A213 (1914): 437-452.
[2] Davies, R.M. “A critical study of Hopkinson pressure bar.” *Philosophical Transactions of the Royal Society of London* A240, no. 821 (1948): 375-457.
[3] Kolsky, H. “An investigation of the mechanical properties of materials at a very high rate of loading.” *Proceedings of the Physical Society of London* B62 (1949): 676-701.
[4] Kolsky, H. *Stress Waves in Solids*. Oxford, Clarendon Press, 1953.
[5] Parry, D.J., P.R. Dixon, S. Hodson, and N. Al-Maliky. “Stress equilibrium effects within Hopkinson bar specimens.” *Journal de Physique IV Proceedings* 04 (C8) (September 1994): C8-107-C8-112.
[6] Gary, G., and H. Zhao. “Dépouillement de l'essai aux barres de Hopkinson par une technique de calcul inverse.” *Journal de Physique IV Proceedings* 04 (C8) (September 1994): C8-89-C8-94.
[7] Zhao, H. *Analysis of the Hopkinson bar test: Application to measuring the dynamic behavior of materials / Analyse de l’essai aux barres d’Hopkinson: Aplication a la mesure du comportement dynamique des materiaux*. PhD Thesis, Ecole Polytehnique, Paris, France, 1992.
[8] DAVID software, Ecole Polytehnique, LMS, Paris, France.
[9] Gavrus, A., J.–P. Le Baron, P. Caestecker, and E. Ragneau. “Investigation of High Speed Behaviour of Ductile Materials by Computer Simulation and Hopkinson Experimental Test.” Paper presented at International Conference on Advances in Mechanical Behaviour, Plasticity and Damage, EUROMAT 2000, Tours, France, November 7-9, 2000.
[10] Gavrus, A., P. Caestecker, E. Ragneau, and B. Davoodi. “Analysis of the dynamic SHPB test using the finite element simulation.” *Journal de Physique IV* 110 (September 2003): 353-358.
[11] Davoodi, B., A Gavrus, and E. Ragneau. “An experimental and numerical analysis of heat transfer problem in SHPB at elevated temperatures.” *Measurement Science and Technology* 16, no. 10 (2005): 2101-2108.
[12] Gavrus, A., B. Davoodi, and E. Ragneau. “A study of material constitutive behaviour at elevated temperature from compressive SHPB test using an inverse analysis method.” *Journal de Physique IV* 134, (August 2006): 661-666. EURODYMAT 2006 - 8th International Conference on Mechanical and Physical Behaviour of Materials under Dynamic Loading.
[13] Rotariu, A. *Contributions regarding the experimental determination of the mechanical properties of some materials in a dynamic regime / Contributii privind determinarea experimentală a proprietăților mecanice ale unor materiale în regim dynamic*. PhD Thesis, Military Technical Academy, Bucharest, 2007.
[14] Diot, S., D. Guines, A. Gavrus, and E. Ragneau. “Two step procedure for identification of metal behaviour from dynamic compression tests.” *International Journal of Impact Engineering* 34 (2007): 1163-1184.
[15] Gavrus, A., F. Bucur, A. Rotariu, and S. Cananau. “Mechanical Behavior Analysis of Metallic Materials using a Finite Element Modeling of the SHPB Test, a Numerical Calibration of the Bar’s Elastic Strains and an Inverse Analysis Method.“ *International Journal of Material Forming* 8, no. 4 (2015): 567-579.
[16] Gavrus, Adinel, Fabien Marco, and Sylvain Guegan. “Experimental Design, Modelling and Numerical Calibration of a High Speed SHPB Mechatronic System Using a Pneumatic Propulsion Device.” Paper presented at the 23rd International Conference on Hydraulics and Pneumatics HERVEX 2017, Baile Govora, Romania, November 8-10, 2017.
[17] Gavrus, Adinel, Sylvain Guegan, and Fabien Marco. “Impact Velocity Control of a Mechatronic Hopkinson Bar System with Robust Pressure Control, Digital Calibration of Strain Wave Measurement and Inverse Analysis.” / “Contrôle des Vitesses d’Impact d’un Système Mécatronique de Barres Hopkinson avec Commande Robuste de la Pression, Calibration Numérique de la Mesure des Ondes de Déformation et Analyse Inverse.” Paper presented at the 24th Congrès Français de Mécanique (CFM 2019), Brest, France, August 26 – 30, 2019.
[18] CAST3M – Finite Element Code, developed by Center of Atomic Energy CEA – France.